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Abstract. The equation for hydrostatic equilibrium has been solved for the equations of state $P = K\rho$ for finite central densities and $K \geq \frac{1}{3}$. The maximum of the mass for an isothermal neutron star core is at $K = 0{\cdot}80$ and not at $K = 1{\cdot}00$. Irrespective of the value of K, the maximum of the core mass occurs at the central density $\rho_c \simeq 3 \times 10^{15}$ g cm^{-3} and the radius of the isothermal core comes out to be 12–13 km.

1. Introduction

Oppenheimer and Volkoff (1939) were the first to discuss the neutron star in the framework of general relativity. They considered a degenerate neutron gas and neglected repulsion due to strong interactions. Parameters of neutron star models for various equations of state are available (Canuto 1975), but still the equations of state at supra nuclear density are poorly known. There is one more way of looking at the structure of neutron stars. A neutron star can be visualised as having an isothermal core with density greater than 2×10^{14} g cm^{-3}. This core is surrounded by a thin crust which makes little contribution towards the total mass of the star. Misner and Zapolsky (1964) were the first to discuss solutions corresponding to isothermal equations of state in the core. Their solutions for the equation $P = K\rho$ correspond to

$$P = K^2/2\pi r^2(K^2 + 6K + 1) \tag{1}$$

and

$$m/r = 2K/(K^2 + 6K + 1) \tag{2}$$

where m is the mass of the core and r is the radius of the core. Brecher and Caporaso (1976) used equations (1) and (2) and showed that the maximum mass of an isothermal neutron star core is $1{\cdot}96\ M_\odot$ for the value $K = 1{\cdot}00$. For the calculation of the mass of the neutron star core they assumed that the density at the core surface was 2×10^{14} g cm^{-3}.

The calculations by Misner and Zapolsky (1964) and Brecher and Caporoso (1976) for an isothermal core pertain only to infinite central density. In this paper, the equations of hydrostatic equilibrium have been solved for the equation of state $P = K\rho$ (for different values of $K \geq \frac{1}{3}$) for finite central densities. By assuming the density at the core boundary, ρ_b, to be 2×10^{14} g cm^{-3} the mass of the neutron star core has been determined for different central densities. For $\rho_c = \infty$, the results are identical to those found by Misner and Zapolsky (1964) and Brecher and Caporoso (1976). Surprisingly, the maximum mass of the neutron star core occurs at the value $K = 0{\cdot}80$ and not at $K = 1{\cdot}00$. The masses of these cores lie between $2{\cdot}1\,M_\odot$ and $2{\cdot}5\,M_\odot$, and the maximum mass value occurs at almost the same central density, $\rho_c \simeq 3\times10^{15}\,\mathrm{g\,cm^{-3}}$.

2. Solutions for an isothermal core

For spherically symmetric, static and non-rotating configurations, the coupled equations to be solved in order to obtain the mass of neutron stars are (Oppenheimer and Volkoff 1939)

$$P = -\tfrac{1}{2}(P+\rho)\nu' \tag{3}$$

$$m = \int 4\pi\rho r^2\,\mathrm{d}r \tag{4}$$

$$P = -(P+\rho)(4\pi P r^3 + m)/r(r-2m) \tag{5}$$

where the prime denotes differentiation with respect to r and ν is the relativistic generalisation of the Newtonian gravitational potential.

The equation of state for an isothermal core is given by

$$P = K\rho. \tag{6}$$

Using equations (3) and (6), we get

$$\mathrm{e}^{-\nu} = A\rho^{2K/(K+1)} \tag{7}$$

where A is a constant depending on the solution of the field equation in the crust.

Equations (4)–(6) lead to

$$1-(8\pi/r)\int_0^{\pi} \rho r^2\,\mathrm{d}r = (8\pi K\rho r^2+1)[1-(2K/K+1)\rho' r/\rho]^{-1}. \tag{8}$$

Equation (8) is computed to evaluate the density and $\mathrm{e}^{-\lambda}$ within the core. For computation we have assumed the central density ρ_c to be

$$\rho_c = 3K/2\pi(K+1)(3K+1)r_0^2 \tag{9}$$

where r_0 is a constant depending on the choice of the central density. The two sides of equation (8) are then matched for various values of r, taken at intervals of $\frac{1}{8}r_0$ from $r = 0$ to $6r_0$, to determine the trends of the various parameters as functions of r. The calculations have been performed for values of $K = \frac{1}{3}$, 0·40, 0·60, 0·80, 0·90, 1·00 and 2·00. Figure 1 shows the variation of m/r with ρ_c/ρ_s. As ρ_c/ρ_s tends to ∞, the value of m/r tends to $2K/(K^2+6K+1)$, the value given by Misner and Zapolsky (1964).

3. Results and discussion

By assuming the density at the core surface, $\rho_b = 2\times10^{14}\,\mathrm{g\,cm^{-3}}$, the mass of the neutron star core has been calculated. Figure 2 shows the variation of mass with central density for different values of K. It is observed that as $\rho_c \to \infty$, the masses approach the value given by Brecher and Caporoso (1976). The maximum mass of the core corresponding to different isothermal equations is plotted with respect to K in figure 3.

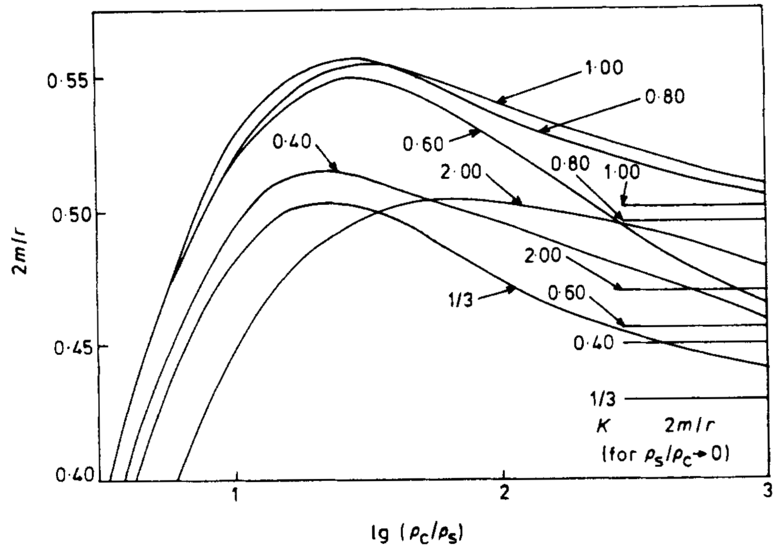

Figure 1. The variation of m/r with ρ_c/ρ_s.

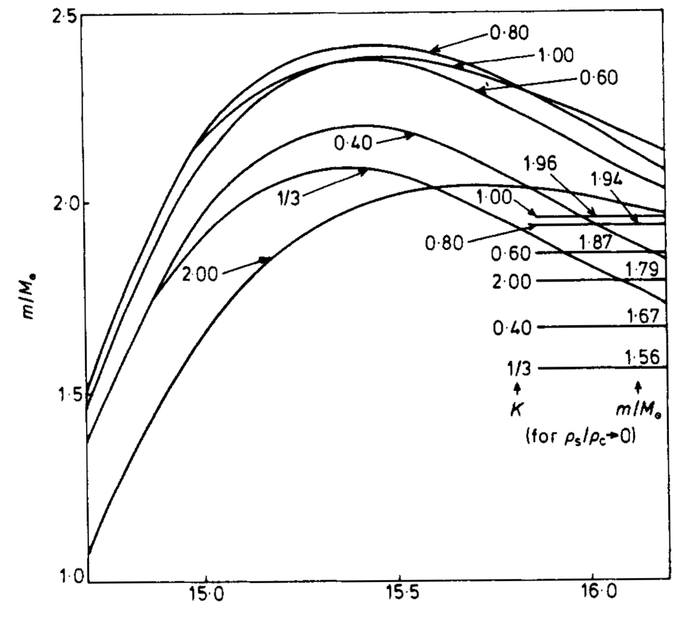

Figure 2. The variation in mass of the neutron star core as a function of central density for various values of K. ρ_c is in g cm^{-3}.

The maximum mass of an isothermal core occurs at the value $K = 0{\cdot}80$. The important conclusions can be summarised as follows.

(i) The maximum of the mass for an isothermal neutron star core has $K = 0{\cdot}80$ and not $K = 1{\cdot}00$. Thus it is not necessary to approach the causality condition to obtain maximum mass for an isothermal neutron star core. We know that various equations of state for supra nuclear densities are known only up to a P/ρ value of approximately 0·8

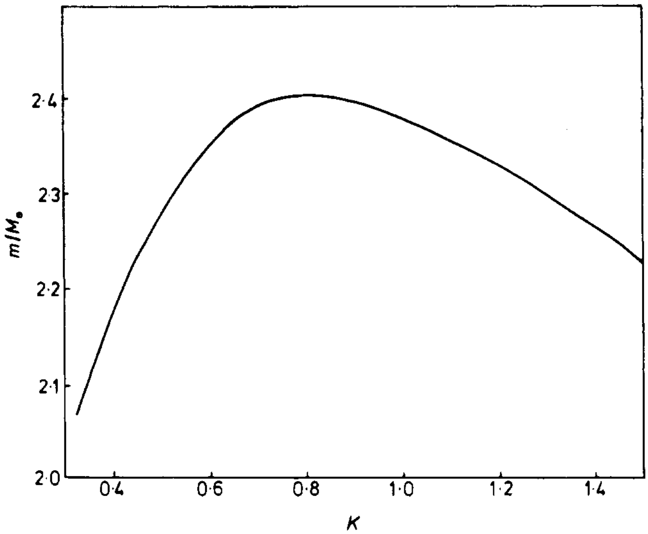

Figure 3. Maximum mass of the core corresponding to different isothermal equations as a function of K.

(e.g. Walecka (1974; $K \simeq 0{\cdot}82$), Bethe and Johnson (1974; $K \simeq 0{\cdot}8$) and Canuto and Chitre (Canuto 1975; $K \simeq 0{\cdot}74$).

(ii) The maximum of the core mass occurs at almost the same central density, $\rho_c \simeq 3 \times 10^{15}\ \mathrm{g\,cm^{-3}}$, irrespective of the value of K chosen. For various equations of state the maximum mass of a neutron star is given by $6{\cdot}0 \times 10^{15} \geqslant \rho_c \geqslant 1{\cdot}2 \times 10^{15}$ (Arnett and Bowers 1977).

(iii) The mass of the isothermal core varies from $2{\cdot}1\ M_\odot$ to $2{\cdot}5\ M_\odot$. Various equations of state also lead to these mass limits (Arnett and Bowers 1977). The radius of the isothermal core is 12–13 km, irrespective of the value of K.

(iv) For determining the maximum value of the mass of a neutron star some previous authors seem to have worked with the tacit assumption that supra luminal speeds of sound (i.e. $\mathrm{d}p/\mathrm{d}\rho > 1$) will eventually lead to a higher mass value. But the maximum mass of the isothermal core occurs at $K = 0{\cdot}8$ and for $K > 0{\cdot}8$ the mass value decreases.

Acknowledgments

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